AQA

Please write clearly in block capitals.

Centre number	
Candidate number	
Surname	
Forename(s)	
Candidate signature	

A-level
MATHEMATICS
Unit Further Pure 3

Wednesday 18 May 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Find the values of the constants a and b for which $ax + b$ is a particular integral of the differential equation

$$2\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 10x$$

[3 marks]

(b) Hence find the general solution of $2\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 10x$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

2 (a) Write down the expansion of $\sin 2x$ in ascending powers of x up to and including the term in x^5.

[1 mark]

(b) It is given that the first non-zero term in the expansion of

$$\sin 2x - 2x(1 - px^2)(1 - x^2)^{-1}$$

in ascending powers of x is qx^5.

Find the values of the rational numbers p and q.

[4 marks]

QUESTION PART REFERENCE

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3 (a) It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = (2x + 1)\ln(x + y)$$

and $$y(0) = 2$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.1$, to obtain an approximation to $y(0.1)$, giving your answer to three decimal places.

[5 marks]

(b) It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (2x + 1)\ln(x + y)$$

and $y = 2$ when $x = 0$.

(i) Use implicit differentiation to find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$, giving your answer in terms of x and y.

[3 marks]

(ii) **Hence** find the first three non-zero terms in the expansion, in ascending powers of x, of $y(x)$. Give your answer in an exact form.

[3 marks]

(iii) Use your answer to part **(b)(ii)** to obtain an approximation to $y(0.1)$, giving your answer to three decimal places.

[1 mark]

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4 (a) The curve with Cartesian equation $\frac{x^2}{c} + \frac{y^2}{d} = 1$ is mapped onto the curve with polar equation $r = \frac{10}{3 - 2\cos\theta}$ by a single geometrical transformation.

By writing the polar equation as a Cartesian equation in a suitable form, find the values of the constants c and d.

[5 marks]

(b) Hence describe the geometrical transformation referred to in part **(a)**.

[1 mark]

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5 (a) Express $\dfrac{1}{(1+x)(2+x)}$ in the form $\dfrac{A}{1+x}+\dfrac{B}{2+x}$, where A and B are integers.

[1 mark]

(b) Use the substitution $u=\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to solve the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{1}{(1+x)(2+x)}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2+x}{1+x}$$

given that $y=1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4$ when $x=0$. Give your answer in the form $y=\mathrm{f}(x)$.

[11 marks]

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6 (a) Use the substitution $a = \dfrac{1}{p}$ to find $\lim\limits_{p \to \infty} \left[\dfrac{\ln p}{p^k} \right]$, where $k > 0$.

[3 marks]

(b) Evaluate the improper integral $\displaystyle\int_1^{\infty} \frac{\ln x}{x^7}\, \mathrm{d}x$, showing the limiting process used.

[4 marks]

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7 Find the solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4y = 10\mathrm{e}^{4x} + 8\sin 2x + 4\cos 2x$$

given that $y = 2.5$ when $x = 0$ and $y = \frac{\pi}{4}$ when $x = \frac{\pi}{4}$.

[10 marks]

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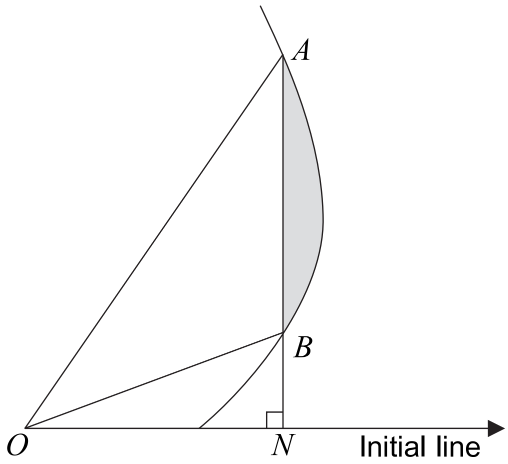
8 The diagram shows the sketch of part of a curve, the pole O and the initial line.



The polar equation of the curve is $r = 1 + \tan\theta$.

The point A is the point on the curve at which $\theta = \dfrac{\pi}{3}$.

The perpendicular, AN, from A to the initial line intersects the curve at the point B.

(a) Find the exact length of OA.

[2 marks]

(b) (i) Given that, at the point B, $\theta = \alpha$, show that $(\cos\alpha + \sin\alpha)^2 = 1 + \dfrac{\sqrt{3}}{2}$.

[4 marks]

(ii) Hence, or otherwise, find α in terms of π.

[2 marks]

(c) Show that the area of triangle OAB is $\dfrac{3 + 2\sqrt{3}}{6}$.

[2 marks]

(d) Find, in an exact simplified form, the area of the shaded region bounded by the curve and the line segment AB.

[7 marks]

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